

EXERCITII PROPUSE

1. Să se calculeze următoarele limite aplicând teoremele asupra operațiilor cu limite de funcții:

a) $\lim_{x \rightarrow 2} (x^2 - 5x + 7)$; b) $\lim_{x \rightarrow 3} (2x^2 e^x - x^3 + 3)$; c) $\lim_{x \rightarrow 0} e^x (x^2 - 2x + 1)$;

d) $\lim_{x \rightarrow \frac{\pi}{4}} (x-1) \sin^2 x$; e) $\lim_{x \rightarrow 1} \frac{x-2}{x^2 + x + 1}$; f) $\lim_{x \rightarrow 2} \sqrt{x^2 - x + 1}$;

g) $\lim_{x \rightarrow 0} (x^2 + 2)^{x+1}$; h) $\lim_{x \rightarrow \infty} (1 + 3x^2)$; i) $\lim_{x \rightarrow -2} e^{3x^2 - x + 1}$;

j) $\lim_{x \rightarrow -\frac{5}{2}} 2^{\frac{2x+5}{x+1}}$; k) $\lim_{x \rightarrow \frac{\pi}{2}} x e^{\operatorname{ctg} x}$; l) $\lim_{x \rightarrow \frac{\pi}{6}} \ln \sin x$;

m) $\lim_{\substack{x \rightarrow 0 \\ x > 0}} \left(\frac{1+x}{2+x} \right)^{\frac{1-\sqrt{x}}{1-x}}$; n) $\lim_{x \rightarrow \infty} a^{3-x-x^2} \ (a > 1)$; p) $\lim_{x \rightarrow \infty} \left(\frac{1}{5} \right)^{x^4 + 2x}$

2. Să se calculeze limitele următoare:

a) $\lim_{x \rightarrow \infty} (x - 12x^2 + x^3)$; b) $\lim_{x \rightarrow -\infty} (x + 2x^2 - 2x^7)$; c) $\lim_{x \rightarrow -\infty} \frac{2x+1}{3x^2 - x + 1}$;

d) $\lim_{x \rightarrow \infty} \frac{3x^3 - 5x^2 + 2}{2 - 3x}$; e) $\lim_{x \rightarrow \infty} \frac{(1-x)(x^2 + 4)}{3x^3 - 2}$; f) $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 1}}{x + 1}$;

g) $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 1}}{x + 1}$; (h) $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 1} + \sqrt{x}}{\sqrt{x^3 + x - x}}$; i) $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 1} + \sqrt{27x^3 - 1}}{\sqrt{x^2 - 1 - x}}$

3. Să se determine:

a) $\lim_{x \rightarrow 4} \frac{x^2 - 6x + 8}{x^2 - 5x + 4}$; b) $\lim_{x \rightarrow 0} \frac{x^4 + 3x^2}{x^5 + x^3 + 2x^2}$; c) $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 7x + 10}$;
d) $\lim_{x \rightarrow 1} \frac{x^m - 1}{x^n - 1}$, $m, n \in \mathbb{Z}$; e) $\lim_{x \rightarrow 1} \frac{x^n - 1 - n(x-1)}{(x-1)^2}$, $n \in \mathbb{N}^*$.

4. Să se determine limitele următoare:

a) $\lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$; b) $\lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{4}{x^2-4} \right)$;
c) $\lim_{x \rightarrow \infty} (\sqrt{x^5+1} - \sqrt[3]{x})$; d) $\lim_{x \rightarrow \infty} (\sqrt{x^3+1} - \sqrt{x+1})$;
e) $\lim_{x \rightarrow \infty} (\sqrt{2x^2+x+1} - \sqrt{x^2+x+1})$; f) $\lim_{x \rightarrow -\infty} (\sqrt{x^4+1} - \sqrt{x^2+2})$;
g) $\lim_{x \rightarrow \infty} (\sqrt{2x^2-3x+5} - \sqrt{2x^2+3x+7})$; h) $\lim_{x \rightarrow \infty} (\sqrt[3]{(x+1)^3+1} - \sqrt[3]{(x-1)^3+1})$;
i) $\lim_{x \rightarrow \infty} x\sqrt{x}(\sqrt{x+1} + \sqrt{x-1} - 2\sqrt{x})$.

5. Să se calculeze limitele:

a) $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+x} - 1}{x}$; b) $\lim_{x \rightarrow -1} \frac{1 + \sqrt[3]{x}}{1 + \sqrt[5]{x}}$; c) $\lim_{x \rightarrow 0} \frac{\sqrt[n]{1+x} - 1}{x}$;
d) $\lim_{x \rightarrow 0} \frac{5x}{\sqrt[3]{1+x} - \sqrt[3]{1-x}}$; e) $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+3x^2} - 1}{x^2 + x^3}$;
f) $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1+x^2}}{\sqrt{1+x} - 1}$; g) $\lim_{x \rightarrow 1} \frac{\sqrt{x} + \sqrt[3]{x} - 2}{x-1}$.

6. Să se calculeze:

a) $\lim_{x \rightarrow \infty} \frac{\ln(x^2 + e^x)}{\ln(x^4 + e^{2x})}$; b) $\lim_{x \rightarrow \infty} \frac{e^x - e^{-x}}{e^x + e^{-x}}$; c) $\lim_{x \rightarrow \infty} \frac{\ln(1+e^x)}{x}$.

7. Să se calculeze:

a) $\lim_{x \rightarrow \infty} (e^x - x)$; b) $\lim_{x \rightarrow \infty} \left(a^x - \frac{1}{a^x} \right)$, $a > 0$;
c) $\lim_{x \rightarrow \infty} \left(\frac{1}{x} + \ln x \right)$; d) $\lim_{x \rightarrow 0} \left(\frac{1}{\sin^2 x} - \frac{1}{4 \sin^2 \frac{x}{2}} \right)$.

8. Să se calculeze:

a) $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 7x}$; b) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$; c) $\lim_{x \rightarrow 0} \frac{\operatorname{tg} \alpha x}{\operatorname{tg} \beta x}$, $\beta \neq 0$;

$$d) \lim_{x \rightarrow 0} \frac{\sqrt{\cos x} - 1}{x^2}; \quad e) \lim_{x \rightarrow 0} \frac{\cos x - \cos 5x}{x^2}; \quad f) \lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{x \sin 2x};$$

$$g) \lim_{x \rightarrow 0} \frac{\sqrt{1+x \sin x} - \sqrt{\cos 2x}}{\operatorname{tg}^2 \frac{x}{2}}; \quad h) \lim_{x \rightarrow 0} \frac{\cos x - \sqrt[3]{\cos x}}{\sin^2 x};$$

$$i) \lim_{x \rightarrow 1} \frac{\operatorname{tg} \sqrt{x^2+1} - \operatorname{tg} \sqrt{x+1}}{\operatorname{tg}(x^2-1)};$$

$$j) \lim_{x \rightarrow \pi} \frac{\cos \frac{x}{2}}{x - \pi};$$

$$k) \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 + \cos 2x}{(\sqrt{\pi} - \sqrt{2x})^2};$$

$$l) \lim_{x \rightarrow -2} \frac{\sin(x^2 + x - 2)}{\sin(x^2 + 3x + 2)};$$

$$m) \lim_{x \rightarrow 0} \frac{\arcsin 3x}{\arcsin x};$$

$$n) \lim_{x \rightarrow \frac{1}{2}} \frac{\arcsin(1-2x)}{\operatorname{tg}(4x^2-1)};$$

9. Să se calculeze:

$$a) \lim_{x \rightarrow 0} \frac{1 + \sin x - \cos x}{1 - \sin x - \cos x};$$

$$b) \lim_{x \rightarrow 0} \frac{1 - \cos x \cos 2x \cos 3x}{1 - \cos x};$$

$$c) \lim_{x \rightarrow 0} \frac{\sin(\sin x)}{x};$$

$$d) \lim_{x \rightarrow -2} \frac{\arcsin(x+2)}{x^2+2x};$$

$$e) \lim_{x \rightarrow \infty} (\sin \sqrt{x+1} - \sin \sqrt{x});$$

$$f) \lim_{h \rightarrow 0} \frac{\operatorname{tg}(x+h) + \operatorname{tg}(x-h) - 2 \operatorname{tg} x}{h^2}.$$

*10. Să se determine:

$$a) \lim_{x \rightarrow 0} \frac{e^{ax} - e^{bx}}{x};$$

$$b) \lim_{x \rightarrow a} \frac{\ln x - \ln a}{x - a}, a > 0;$$

$$c) \lim_{x \rightarrow a} \frac{a^x - a^a}{x - a}, a > 0;$$

$$d) \lim_{x \rightarrow a} \frac{x^x - a^a}{x - a}, a > 0;$$

$$e) \lim_{x \rightarrow a} \frac{\log_a[1 + (x-a)]}{x - a}, a > 0, a \neq 1;$$

$$f) \lim_{x \rightarrow a} \frac{a^{\sin x} - a^{\sin a}}{x - a}, a > 0;$$

$$g) \lim_{x \rightarrow 0} \frac{\ln(1 + \arcsin x)}{x};$$

$$h) \lim_{x \rightarrow \frac{\pi}{4}} \frac{2^{\operatorname{tg} x} - 2}{x - \frac{\pi}{4}};$$

$$l) \lim_{x \rightarrow 1} \frac{\ln(x^2 - 3x + 3)}{\sin(x^2 - 5x + 4)}.$$

11. Să se determine limitele:

$$a) \lim_{x \rightarrow \infty} \left(1 + \frac{2x}{x^2+1}\right)^{x+2};$$

$$b) \lim_{x \rightarrow \infty} \left(\frac{3x+2}{3x+1}\right)^{x+1};$$

$$c) \lim_{\substack{x \rightarrow 1 \\ x > 1}} \left(\frac{1 + \sqrt{x-1}}{\sqrt{2x-1}}\right)^{\frac{1}{\sqrt{x-1}}};$$

$$d) \lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3}\right)^{\frac{1}{x}}, a, b, c > 0;$$

$$e) \lim_{x \rightarrow 0} (e^x + x)^{\frac{1}{x}};$$

$$f) \lim_{x \rightarrow \infty} \left(\ln \frac{e^x}{x+1}\right)^{2x};$$

$$g) \lim_{x \rightarrow 0} (1 + \operatorname{tg} x)^{\operatorname{ctg} x};$$

$$\text{h) } \lim_{x \rightarrow 0} \left(\frac{1 + \operatorname{tg} x}{1 + \sin x} \right)^{\frac{1}{x^3}} ;$$

$$\text{i) } \lim_{x \rightarrow \frac{\pi}{4}} (\operatorname{ctg} x)^{\operatorname{tg} 2x} ;$$

$$\text{j) } \lim_{x \rightarrow \infty} \left(\cos \frac{a}{x} \right)^x ;$$

***12.** Să se determine limitele:

$$\text{a) } \lim_{\substack{x \rightarrow 0 \\ x > 0}} x^{\sin x} ;$$

$$\text{b) } \lim_{\substack{x \rightarrow 0 \\ x > 0}} (\sin x)^x ;$$

$$\text{c) } \lim_{\substack{x \rightarrow 1 \\ x < 1}} (1-x)^{\frac{\pi}{2} - \arcsin x} ;$$

$$\text{d) } \lim_{x \rightarrow \infty} x^{\frac{\sin \frac{1}{x}}{x}} ;$$

$$\text{e) } \lim_{x \rightarrow 1} \left(\frac{1}{x-1} \right)^{x-1} ;$$

$$\text{f) } \lim_{x \rightarrow \infty} (1+x)^{\frac{1}{x}} .$$